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Unsteady MHD convection flow of polar fluids past a vertical moving porous plate in a porous medium

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Abstract

The objectives of the present study are to investigate the unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting polar fluid via a porous medium past a semi-infinite vertical porous moving plate in the presence of a transverse magnetic field. The plate moves with a constant velocity in the longitudinal direction, and the free stream velocity follows an exponentially increasing or decreasing small perturbation law. A uniform magnetic field acts perpendicularly to the porous surface which absorbs the polar fluid with a suction velocity varying with time. The effects of material parameters on the velocity and temperature fields across the boundary layer are investigated. The method of solution can be applied for small perturbation approximation. Numerical results of velocity distribution of polar fluids are compared with the corresponding flow problems for a Newtonian fluid. For a constant plate moving velocity with the given magnetic and permeability parameters, and Prandtl and Grashof numbers, the effect of increasing values of suction velocity parameter results in an increasing surface skin friction. It is also observed that the surface skin friction decreases by increasing the plate moving velocity. © 2001 Published by Elsevier Science Ltd.

1. Introduction

The study of flow and heat transfer for an electrically conducting polar fluid past a porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as magnetohydrodynamic (MHD) generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics [1]. Polar fluids are fluids with microstructure belonging to a class of fluids with non-symmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium [2–4]. A great number of Darcian porous MHD studies have been carried out examining the effects of magnetic field on hydrodynamic flow without heat transfer in various configurations, e.g., in channels and past plates and wedges, etc. [5,6].

Gribben [7] considered the MHD boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of a pressure gradient. He has obtained solutions for large and small magnetic Prandtl numbers using the method of matched asymptotic expansion. Takhar and Ram [8] studied the effects of Hall currents on hydromagnetic free convection boundary layer flow via a porous medium past a plate, using harmonic analysis. Takhar and Ram [9] also studied the MHD free porous convection heat transfer of water at 4°C through a porous medium.

Soundalgekar [10] obtained approximate solutions for the two-dimensional flow of an incompressible, viscous fluid past an infinite porous vertical plate with constant suction velocity normal to the plate, the difference between the temperature of the plate and the free stream is moderately large causing the free convection currents.

Raptis and Kafousias [11] studied the influence of a magnetic field upon the steady free convection flow through a porous medium bounded by an infinite vertical plate with a constant suction velocity, and when the plate temperature is also constant. Raptis [12] studied

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Nomenclature

A	suction velocity parameter
B_0	magnetic flux density
C_{f}	skin friction coefficient
C_{p}	specific heat at constant pressure
G	Grashof number
g	acceleration due to gravity
K	permeability of the porous medium
k	thermal conductivity
M	magnetic field parameter
N	dimensionless material parameter
Nu	Nusselt number
n	dimensionless exponential index
Pr	Prandtl number
T	temperature
t	dimensionless time
U_0	scale of free stream velocity
u, v	components of velocities along and perpendicular to the plate, respectively
V_0	scale of suction velocity
x, y	distances along and perpendicular to the plate, respectively

Greek symbols

α	fluid thermal diffusivity
β	dimensionless viscosity ratio
β_{f}	coefficient of volumetric expansion of the working fluid
γ	spin-gradient viscosity
ε	scalar constant ($\ll 1$))
σ	electrical conductivity
ρ	fluid density
Λ	coefficient of gyro-viscosity
μ	fluid dynamic viscosity
v	fluid kinematic viscosity
v_{r}	fluid kinematic rotational viscosity
θ	dimensionless temperature
ω	angular velocity vector

Superscripts

$'$	differentiation with respect to y
$*$	dimensional properties

Subscripts

p	plate
w	wall condition
∞	free stream condition

mathematically the case of time-varying two-dimensional natural convective heat transfer of an incompressible, electrically conducting viscous fluid via a highly porous medium bounded by an infinite vertical porous plate.

However, most of the previous works assume that the plate is at rest. In the present work, we consider the case of a semi-infinite moving porous plate with a constant velocity in the longitudinal direction when the magnetic field is imposed transversely to the plate. We also consider the free stream to consist of a mean velocity and temperature with a superimposed exponentially variation with time.

In general, the study of Darcian porous MHD is very complicated. It is necessary to consider in detail the distribution of velocity and temperature distributions across the boundary layer, in addition to the surface skin friction. The present work is an attempt to shed some light on these issues.

2. Formulation

We consider the two-dimensional unsteady flow of a laminar, incompressible fluid past a semi-infinite vertical porous moving plate embedded in a porous medium and subjected to a transverse magnetic field in the presence of a pressure gradient. The physical model and geometrical coordinates are shown in Fig. 1. It is assumed

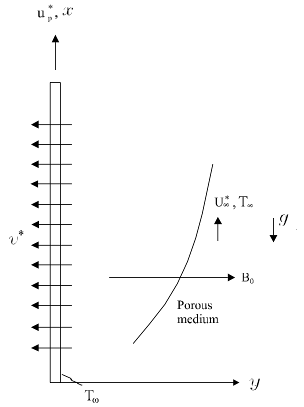

Fig. 1. Physical model and coordinate system of problem.

that there is no applied voltage which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible [13]. Viscous and Darcy's resistance terms are taken into account with constant permeability of the porous medium. The MHD term is derived from an order-of-magnitude analysis of the full Navier–Stokes equations. It is assumed here that the hole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. We regard the porous medium as an assemblage of small identical spherical particles fixed in space, following Yamamoto and Iwamura [14]. Due to the semi-infinite plane surface assumption, furthermore, the flow variables are functions of y^* and t^* only.

Under these conditions, the governing equations, i.e., the mass, momentum and energy conservation equations can be written in a Cartesian frame of reference, as:

continuity:

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

linear momentum:

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + (\nu + \nu_r)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f(T - T_\infty) - \nu\frac{u^*}{K^*} - \frac{\sigma}{\rho}B_0^2 u^* + 2\nu_r \frac{\partial \omega^*}{\partial y^*}, \tag{2}$$

angular momentum:

$$\rho j^*\left(\frac{\partial \omega^*}{\partial t^*} + v^*\frac{\partial \omega^*}{\partial y^*}\right) = \gamma \frac{\partial^2 \omega^*}{\partial y^{*2}}, \tag{3}$$

energy:

$$\frac{\partial T}{\partial t^*} + v^*\frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}}, \tag{4}$$

where x^* and y^* are the dimensional distances longitudinal and perpendicular to the plate, respectively, u^*,v^* the components of dimensional velocities along the x^* and y^* directions, respectively, ρ the density and ν the kinematic viscosity, ν_r the kinematic rotational viscosity, g the acceleration of gravity, β_f the coefficient of volumetric thermal expansion of the fluid, K^* the permeability of the porous medium, σ the electrical conductivity of the fluid, B_0 the magnetic induction, j^* the micro-inertia density, ω^* the component of the angular velocity vector normal to the xy-plane, γ the spin-gradient viscosity, T the temperature, and α is the effective fluid thermal diffusivity.

The third term on the RHS of the momentum Eq. (2) denotes buoyancy effects, the fourth is the bulk matrix linear resistance, i.e., Darcy term, the fifth is the MHD term. The heat due to viscous dissipation is neglected for small velocities in Eq. (4). Also, Darcy dissipation term is neglected because it is of the same order-of-magnitude as the viscous dissipation term. It is assumed that the porous plate moves with constant velocity (u_p^*) in the longitudinal direction, and the free stream velocity (U_∞^*) follows an exponentially increasing or decreasing small perturbation law. We also assume that the plate temperature (T) and suction velocity (v^*) vary exponentially with time.

Under these assumptions, the appropriate boundary conditions for the velocity and temperature fields are

$$u^* = u_p^*, \quad T = T_w + \varepsilon(T_w - T_\infty)e^{n^*t^*},$$
$$\frac{\partial \omega^*}{\partial y^*} = -\frac{\partial^2 u^*}{\partial y^{*2}} \ \text{at} \ y^* = 0, \tag{5}$$

$$u^* \to U_\infty^* = U_0\left(1 + \varepsilon e^{n^*t^*}\right),$$
$$T \to T_\infty, \quad \omega^* \to 0 \ \text{as} \ y^* \to \infty \tag{6}$$

in which n^* is a scalar constant, and U_0 is a scale of free stream velocity.

From the continuity equation (1), it is clear that the suction velocity normal to the plate is a function of time only and we shall take it in the form:

$$v^* = -V_0\left(1 + \varepsilon A e^{n^*t^*}\right), \tag{7}$$

where A is a real positive constant ε and εA small less than unity and V_0 is a scale of suction velocity which has non-zero positive constant. Outside the boundary layer, Eq. (2) gives

$$-\frac{1}{\rho}\frac{dp^*}{dx^*} = \frac{dU_\infty^*}{dt^*} + \frac{\nu}{K^*}U_\infty^* + \frac{\sigma}{\rho}B_0^2 U_\infty^*. \tag{8}$$

We now introduce the dimensionless variables, as follows:

$$u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{V_0}, \quad y = \frac{V_0 y^*}{\nu}, \quad U_\infty = \frac{U_\infty^*}{U_0}, \quad U_p = \frac{u_p^*}{U_0},$$
$$\omega = \frac{\nu}{U_0 V_0}\omega^*, \quad t = \frac{t^* V_0^2}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad n = \frac{n^*\nu}{V_0^2},$$
$$K = \frac{K^* V_0^2}{\nu^2}, \quad j = \frac{V_0^2}{\nu^2}j^*,$$

$Pr = \dfrac{\nu\rho C_p}{k} = \dfrac{\nu}{\alpha}$ is the Prandtl number,

$M = \dfrac{\sigma B_0^2 \nu}{\rho V_0^2}$ is the magnetic field parameter,

$G = \dfrac{\nu\beta_f g(T_w - T_\infty)}{U_0 V_0^2}$ is the Grashof number. (9)

Furthermore, the spin-gradient viscosity γ which gives some relationship between the coefficients of viscosity and micro-inertia, is defined as

$$\gamma = \left(\mu + \frac{\Lambda}{2}\right)j^* = \mu j^*\left(1 + \frac{1}{2}\beta\right), \tag{10}$$

where β denotes the dimensionless viscosity ratio, defined as follows:

$$\beta = \frac{\Lambda}{\mu} \tag{11}$$

in which Λ is the coefficient of gyro-viscosity (or vortex viscosity).

In view of Eqs. (7)–(11), the governing equations (2)–(4) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + (1 + \beta)\frac{\partial^2 u}{\partial y^2} + G\theta + N(U_\infty - u) + 2\beta\frac{\partial \omega}{\partial y}, \tag{12}$$

$$\frac{\partial\omega}{\partial t} - (1 + \varepsilon A\,e^{nt})\frac{\partial\omega}{\partial y} = \frac{1}{\eta}\frac{\partial^2\omega}{\partial y^2}, \quad (13)$$

$$\frac{\partial\theta}{\partial t} - (1 + \varepsilon A\,e^{nt})\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}, \quad (14)$$

where

$$N = \left(M + \frac{1}{K}\right), \quad \eta = \frac{\mu j^*}{\gamma} = \frac{2}{2+\beta}.$$

The boundary conditions (5) and (6) are then given by the following dimensionless form:

$$u = U_p, \quad \theta = 1 + \varepsilon e^{nt}, \quad \frac{\partial\omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2} \quad \text{at } y = 0, \quad (15)$$

$$u \to U_\infty, \quad \theta \to 0, \quad \omega \to 0 \quad \text{as } y \to \infty. \quad (16)$$

3. Solution

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we may represent the linear and angular velocities and temperature as

$$u = u_0(y) + \varepsilon e^{nt}u_1(y) + O(\varepsilon^2) + \cdots \quad (17)$$

$$\omega = \omega_0(y) + \varepsilon e^{nt}\omega_1(y) + O(\varepsilon^2) + \cdots \quad (18)$$

$$\theta = \theta_0(y) + \varepsilon e^{nt}\theta_1(y) + O(\varepsilon^2) + \cdots \quad (19)$$

Substituting Eqs. (17)–(19) in Eqs. (12)–(14) and equating the harmonic and non-harmonic terms, neglecting the coefficient of $O(\varepsilon^2)$, we get the following pairs of equations for $(u_0, \omega_0, \theta_0)$ and $(u_1, \omega_1, \theta_1)$.

$$(1+\beta)u_0'' + u_0' - Nu_0 = -N - G\theta_0 - 2\beta\omega_0', \quad (20)$$

$$(1+\beta)u_1'' + u_1' - (N+n)u_1 = -(N+n) - Au_0' - G\theta_1 - 2\beta\omega_1', \quad (21)$$

$$\omega_0'' + \eta\omega_0' = 0, \quad (22)$$

$$\omega_1'' + \eta\omega_1' - n\eta\omega_1 = -A\eta\omega_0', \quad (23)$$

$$\theta_0'' + Pr\theta_0' = 0, \quad (24)$$

$$\theta_1'' + Pr\theta_1' - nPr\theta_1 = APr\theta_0'. \quad (25)$$

Here primes denote differentiation with respect to y. The corresponding boundary conditions can be written as:

$$u_0 = U_p, \quad u_1 = 0, \quad \omega_0' = -u_0'', \quad \omega_1' = -u_1'', \quad \theta_0 = 1, \quad \theta_1 = 1 \quad \text{at } y = 0 \quad (26)$$

$$u_0 = 1, \quad u_1 = 1, \quad \omega_0 \to 0, \quad \omega_1 \to 0, \quad \theta_0 \to 0, \quad \theta_1 \to 0 \quad \text{as } y \to \infty. \quad (27)$$

The solutions of Eqs. (20)–(25) with satisfying boundary conditions (26) and (27) are given by

$$u_0(y) = 1 + a_1 e^{-h_2 y} + a_2 e^{-Pry} + a_3 e^{-\eta y}, \quad (28)$$

$$u_1(y) = 1 + b_1 e^{-h_1 y} + b_2 e^{-h_2 y} + b_3 e^{-h_3 y} + b_4 e^{-h_4 y} + b_5 e^{-Pry} + b_6 e^{-\eta y}, \quad (29)$$

$$\omega_0(y) = c_1 e^{-\eta y}, \quad (30)$$

$$\omega_1(y) = c_2 e^{-h_1 y} - \frac{A\eta}{n}c_1 e^{-\eta y}, \quad (31)$$

$$\theta_0(y) = e^{-Pry}, \quad (32)$$

$$\theta_1(y) = e^{-h_4 y} + \frac{A}{n}Pr\left(e^{-h_4 y} - e^{-Pry}\right), \quad (33)$$

where

$$h_1 = \frac{\eta}{2}\left[1 + \sqrt{1 + \frac{4n}{\eta}}\right],$$

$$h_2 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4N(1+\beta)}\right],$$

$$h_3 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4(N+n)(1+\beta)}\right],$$

$$h_4 = \frac{Pr}{2}\left(1 + \sqrt{1 + \frac{4n}{Pr}}\right),$$

and

$$a_1 = U_p - 1 - a_2 - a_3,$$

$$a_2 = -\frac{G}{(1+\beta)Pr^2 - Pr - N},$$

$$a_3 = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta - N}c_1,$$

$$b_1 = \frac{2\beta h_1}{(1+\beta)h_1^2 - h_1 - (N+n)}c_2,$$

$$b_2 = -\frac{Ah_2}{n}a_1,$$

$$b_3 = \frac{2\beta h_1}{(1+\beta)h_1^2 + (2\beta-1)h_1 - 2\beta h_3^2 - (N+n)} \times \left[\frac{(1+3\beta)h_1^2 - h_1 - (N+n)}{2\beta h_1}k_2 + \frac{k_1}{h_1}\right],$$

$$b_4 = -G\left(1 + \frac{APr}{n}\right)\frac{1}{(1+\beta)h_4^2 - h_4 - (N+n)},$$

$$b_5 = \frac{APrG}{n}\frac{1}{(1+\beta)Pr^2 - Pr - N},$$

$$b_6 = \frac{A\eta a_3 - \{\frac{2\beta A\eta^2}{n}\}c_1}{(1+\beta)\eta^2 - \eta - (N+n)},$$

$$c_1 = \frac{(1+\beta)\eta - 1 - \{N/\eta\}}{(1-\beta)\eta^2 - \eta - N + 2\beta h_2^2} \times \left[(U_p - 1)h_2^2 + \left(Pr^2 - h_2^2\right)a_2\right],$$

$$c_2 = -\frac{1}{h_1}\left[k_1 + b_1 h_1^2 + b_3 h_3^2\right],$$

$$k_1 = -\frac{A\eta^2}{n}c_1 + b_2 h_2^2 + b_4 h_4^2 + b_5 Pr^2 + b_6\eta^2,$$

$$k_2 = -(1 + b_2 + b_4 + b_5 + b_6).$$

By virtue of Eqs. (17)–(19), we obtain the streamwise and angular velocities and temperature, as follows:

$$u(y,t) = 1 + a_1 e^{-h_2 y} + a_2 e^{-Pry} + a_3 e^{-\eta y} + \varepsilon e^{nt}\left[1 + b_1 e^{-h_1 y} + b_2 e^{-h_2 y} + b_3 e^{-h_3 y} + b_4 e^{-h_4 y} + b_5 e^{-Pry} + b_6 e^{-\eta y}\right], \quad (34)$$

$$\omega(y,t) = c_1 e^{-\eta y} + \varepsilon e^{nt}\left[c_2 e^{-h_1 y} - \frac{A\eta}{n} c_1 e^{-\eta y}\right], \quad (35)$$

$$\theta(y,t) = e^{-Pry} + \varepsilon e^{nt}\left[e^{-h_4 y} + \frac{A}{n} Pr\left(e^{-h_4 y} - e^{-Pry}\right)\right]. \quad (36)$$

Given the velocity field in the boundary layer, we can now calculate the skin friction at the wall of the plate, which is given by

$$\tau_w = \frac{\tau_w^*}{\rho U_0 V_0} = \left.\frac{\partial u}{\partial y}\right|_{y=0} = (1 - U_p)h_2 + (h_2 - Pr)a_2 + (h_2 - \eta)a_3 - \varepsilon e^{nt}[b_1 h_1 + b_2 h_2 + b_3 h_3 + b_4 h_4 + b_5 Pr + b_6 \eta]. \quad (37)$$

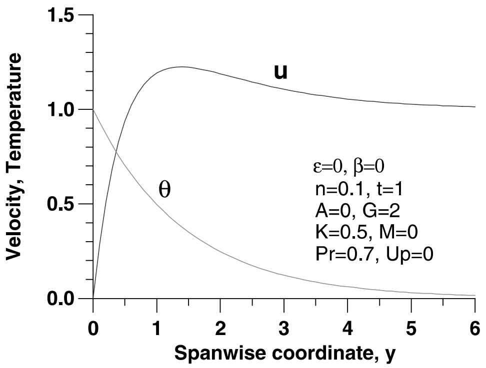

Fig. 2. Distributions of velocity and temperature profiles of Newtonian fluid across the boundary layer for a stationary vertical porous plate in the absence of magnetic field.

We can also calculate the heat transfer coefficient in terms of the Nusselt number, as follows:

$$Nu = x\frac{(\partial T/\partial y^*)_w}{T_w - T_\infty}, \quad (38)$$

$$Nu Re_x^{-1} = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = -Pr + \varepsilon e^{nt}\left[\frac{A}{n}Pr^2 - h_4\left(1 + \frac{A}{n}Pr\right)\right], \quad (39)$$

where $Re_x = V_0 x/\nu$ is the Reynolds number.

4. Results and discussion

The formulation of the effect of magnetic fields and suction velocity varying exponentially with time about a non-zero constant mean value on the flow and heat transfer of an incompressible polar conducting fluid along a semi-infinite vertical porous moving plate has been carried out in the preceding sections. This enables us to carry out the numerical computations for the velocity and temperature for various values of the flow and material parameters. In the present study the boundary condition for $y \to \infty$ is replaced by identical ones at y_{max} which is a sufficiently large value of y where the velocity profile u approaches the relevant free stream velocity. We choose $y_{max} = 6$ and a step size $\Delta y = 0.001$.

The general distributions of velocity and temperature profiles of Newtonian fluids across the boundary layer for a stationary porous plate in the case of absence of magnetic field are shown in Fig. 2.

In Figs. 3–10 we have prepared some graphs of the velocity and angular velocity profiles for polar fluids with the fixed flow and material parameters, $n, t, A, \epsilon, M, K, G, Pr$ and U_p which are listed in the figure captions. The effect of viscosity ratio β on the velocity and angular velocity for a stationary porous plate in the absence of magnetic field is presented in Fig. 3. The numerical results show that the velocity distribution is lower for a

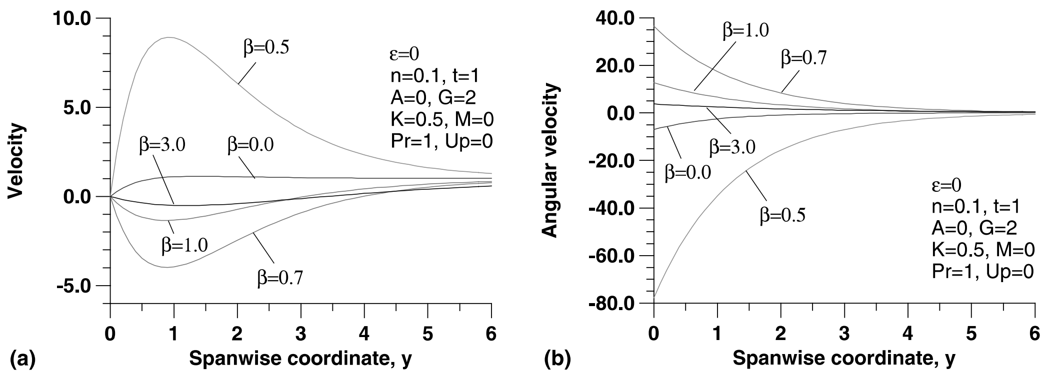

Fig. 3. Velocity and angular velocity profiles against spanwise coordinate y for different values of viscosity ratio β.

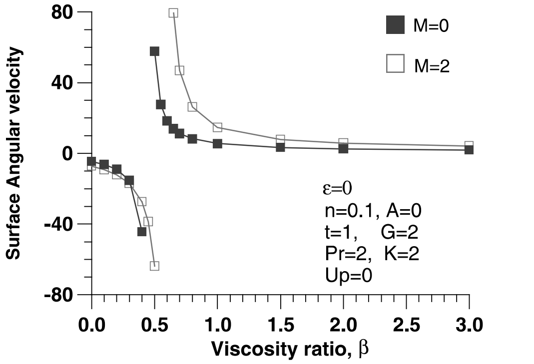

Fig. 4. Surface angular velocity versus viscosity ratio for different values of magnetic parameter M.

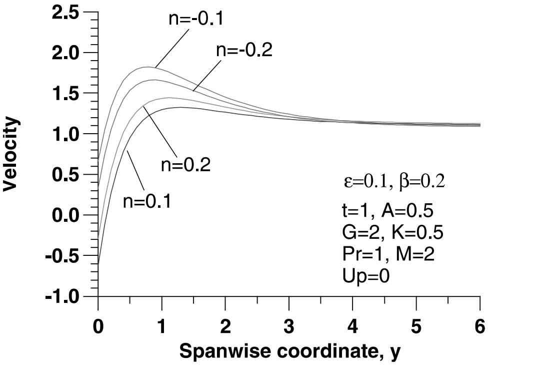

Fig. 5. Velocity profiles against spanwise coordinate y for different values of dimensionless exponential index n .

Newtonian fluid ($\beta = 0$) with the fixed flow and material parameters, as compared with a polar fluid when the viscosity ratio is less than 0.5. When β takes values larger than 0.5, however, the velocity distribution shows a decelerating nature near the porous plate. In addition, the angular velocity distributions do not show consistent variations with increment of viscosity ratio parameter.

In order to elucidate these behaviors, we calculate the surface angular velocity on the stationary porous plate versus viscosity ratio β for different values of the magnetic parameter M in Fig. 4, where it is seen that the critical value of viscosity ratio exists. This value tends to increase as the magnetic field parameter increases.

For the case of a stationary porous plate, velocity profiles against the spanwise coordinate y for different values of dimensionless exponential index n are shown in Fig. 5. It is seen that when approaching $n \to 0^-$, the magnitude of the velocity distribution across the boundary layer increases, and then decays to the relevant free stream velocity. However, the velocity distribution decreases as the exponential index n approaches the $n \to 0^+$ value.

Fig. 6 illustrates the variation of velocity and angular velocity distribution across the boundary layer for various values of the plate velocity U_p in the direction of fluid flow. The peak value of velocity across the boundary layer decreases near the porous plate as the plate velocity increases. However, the values of angular velocity on the porous plate are increased as the plate velocity increases.

For different values of the magnetic field parameter M, the velocity and angular velocity profiles are plotted in Fig. 7. It is obvious that the effect of increasing values of magnetic field parameter results in a decreasing velocity distribution across the boundary layer. Furthermore, the results show that the values of angular velocity on the porous plate are decreased as M increases.

Fig. 8 shows the velocity profiles for different values of the permeability parameter K. Clearly as K increases the velocity boundary layer tends to decrease, and then decays to the relevant free stream velocity.

The velocity and angular velocity profiles against spanwise coordinate$y$ for different values of Grashof number G are described in Fig. 9. It is observed that an

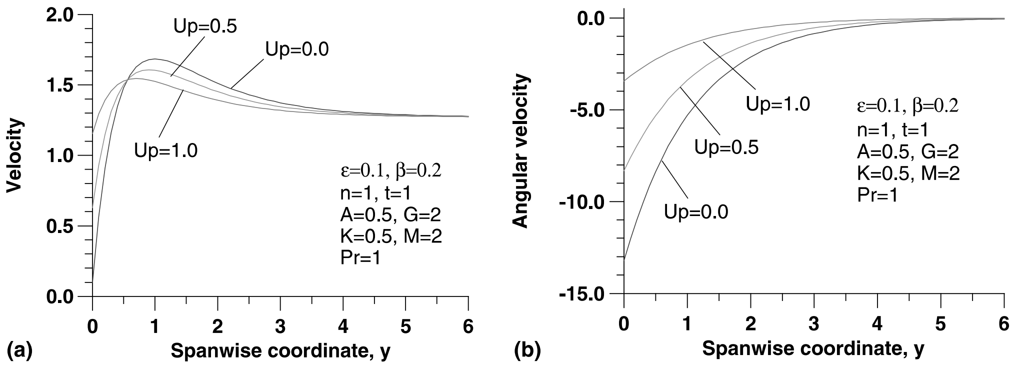

Fig. 6. Velocity and angular velocity profiles against spanwise coordinate y for different values of plate moving velocity U_p.

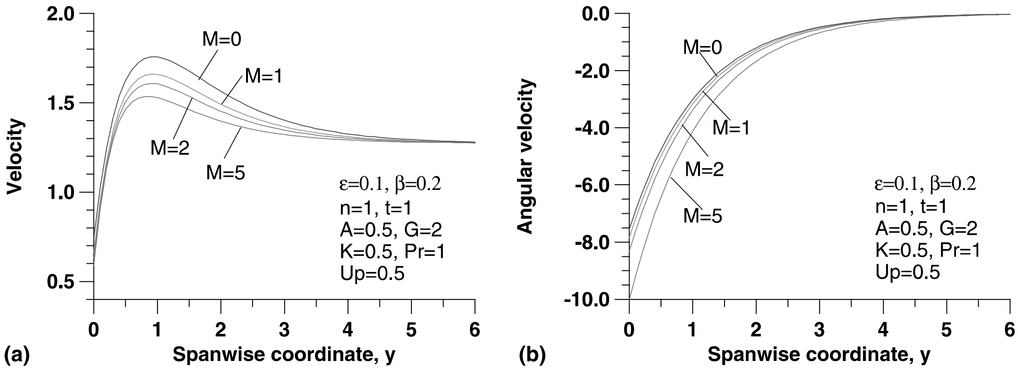

Fig. 7. Velocity and angular velocity profiles against spanwise coordinate y for different values of magnetic parameter M.

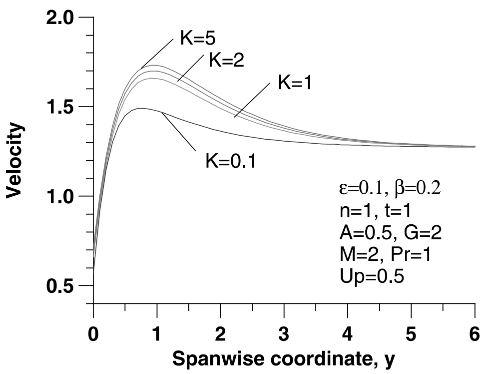

Fig. 8. Velocity profiles against spanwise coordinate y for different values of permeability K.

increase in G leads to a rise in the values of velocity, but decreases due to angular velocity. Here the positive value of G corresponds to a cooling of the surface by natural convection. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as the Grashof number increases, and then decays to the free stream velocity.

Fig. 10(a) shows the velocity profiles against spanwise coordinate y for different values of Prandtl number Pr. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity, and then approaches a constant value of about 1.3 which is relevant to the free stream velocity at the edge of boundary layer. The results also reveal that the peak value of velocity decreases as Pr decreases.

Typical variations of the temperature profiles along the spanwise coordinate are shown in Fig. 10(b) for different values of Prandtl number Pr . The numerical results show that an increase of Prandtl number results in a decreasing thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr. Hence the boundary layer is thicker and the rate of heat transfer is reduced, for gradients have been reduced.

As shown in Fig. 11, it has been observed that for a constant suction velocity parameter A with given flow and material parameters, the effect of increasing values of plate moving velocity U_p results in a decreasing

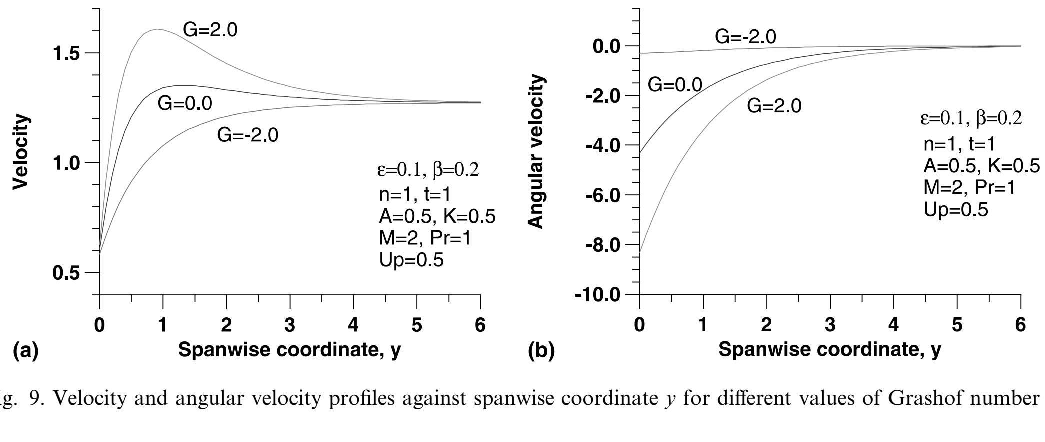

Fig. 9. Velocity and angular velocity profiles against spanwise coordinate y for different values of Grashof number G.

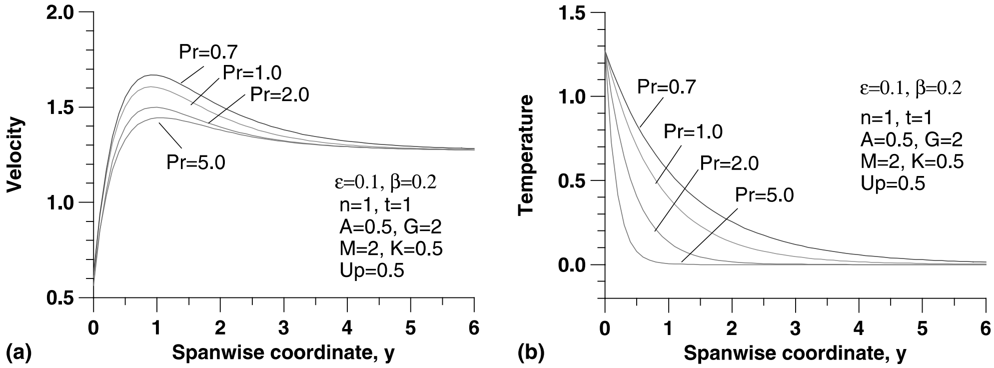

Fig. 10. Velocity and temperature profiles against spanwise coordinate y for different values of Prandtl number Pr.

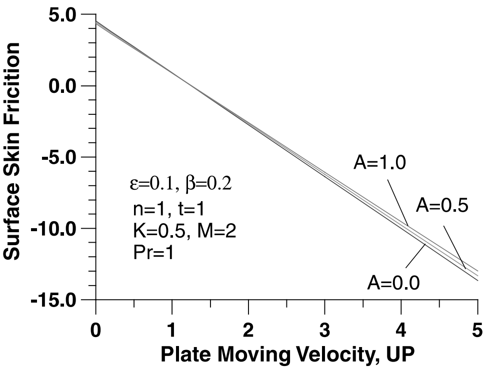

Fig. 11. Variation of the surface skin friction with the plate moving velocity U_p for various suction velocity parameters A.

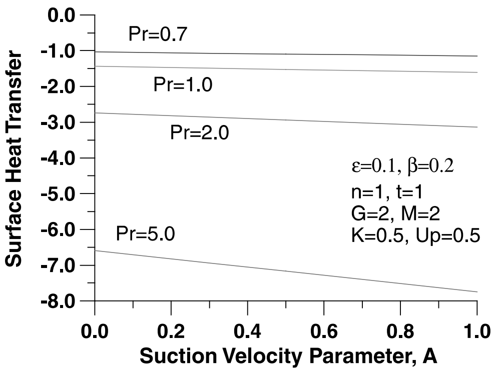

Fig. 12. Variation of the surface heat transfer with the suction velocity parameter A for different values of Prandtl number Pr.

surface skin friction on the porous plate. It is difficult to show clearly the corresponding profiles of surface skin friction due to very little variation. It is also evident that for different values of the suction velocity parameter A, the surface skin friction has zero value near $U_p = 1.2$.

Fig. 12 illustrates the variation of surface heat transfer with the suction velocity parameter A for several values of Prandtl number. Numerical results show that for given flow and material parameters which are listed in the figure caption, the surface heat transfer from the porous plate tends to decrease slightly on increasing the magnitude of suction velocity.

5. Conclusions

We have examined the governing equations for an unsteady, incompressible polar fluid past a semi-infinite porous moving plate whose velocity is maintained at a constant value, and embedded in a porous medium and subjected to the presence of a transverse magnetic field. The method of solution can be applied for small perturbation approximation. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on the material parameters. We observe that, when the magnetic parameter increases the velocity decreases, whereas when the permeability parameter or Grashof number increases the velocity increases.

It is recognized that there are many other methods that could be considered in order to describe some reasonable solutions for this particular type of problem. For a better understanding of the thermal behavior of this work, however, it may be necessary to perform the experimental works. In the near future, we would be glad to compare these theoretical results with those obtained by anyone in the same field.

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